VARIABLES CONTROL CHARTS

Example 1

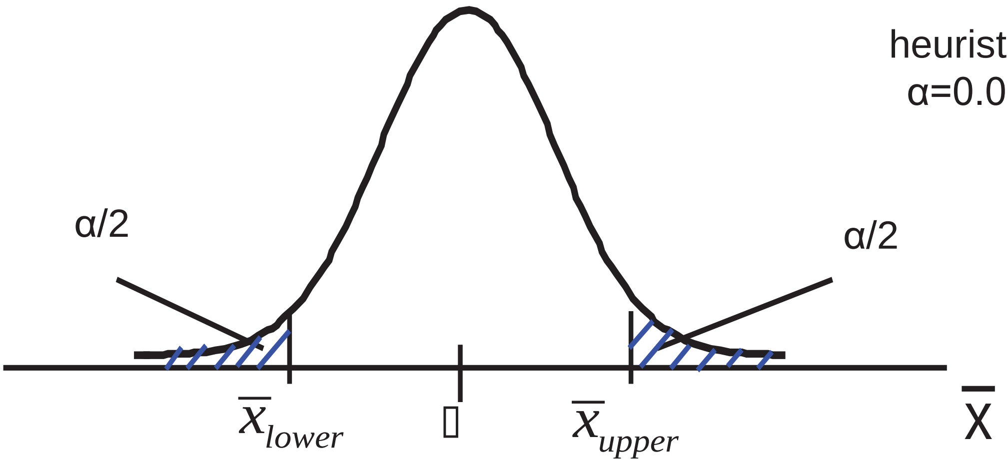
The assumed expected value of the mass of packages produced by an automatic machine is 250 g, the known variance of the process is 1 g^2.
The mean of the sample of 5 elements taken from the process is:

$$\bar{x} = 249.6 \text{ g}$$

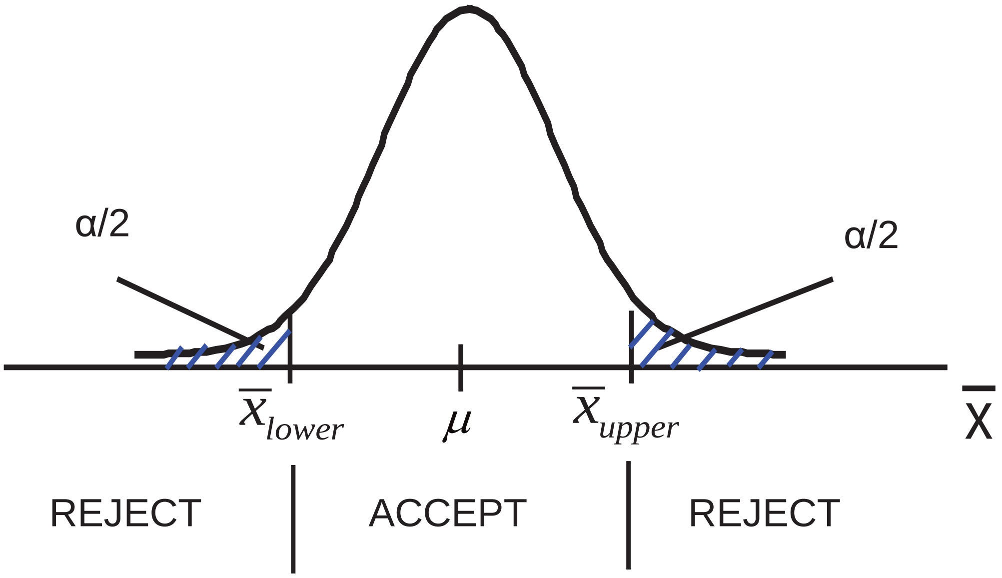
Do we believe that the expected value of the mass of packages is 250 g?

$$P(\underbrace{\mu - z_{\alpha/2}\,\sigma/\sqrt{n} < \overline{x} < \mu + z_{\alpha/2}\,\sigma/\sqrt{n}}) = 1-\alpha$$

α: probability of having the average out of this range.



heuristic:
α=0.05



If the average is in the accept/reject region we accept/reject, that the expected value is 250g.

The region of acceptance:

$$\mu - z_{\alpha/2}\,\sigma\big/\sqrt{n} < \overline{x} < \mu + z_{\alpha/2}\,\sigma\big/\sqrt{n}$$

$$UCL = \overline{x}_{\text{upper}} = \mu + z_{\alpha/2}\sigma/\sqrt{n} =$$

$$z_{\alpha/2} =$$

$$LCL = \overline{x}_{\text{lower}} = \mu - z_{\alpha/2}\sigma/\sqrt{n} =$$

Decision:

The region of acceptance:

$$\mu - z_{\alpha/2}\,\sigma\big/\sqrt{n} < \overline{x} < \mu + z_{\alpha/2}\,\sigma\big/\sqrt{n}$$

Take samples (subgroup) time to time and plot their mean as a function of time!

→ control chart

- in statistical control: continue
- out of control: stop the process

The intervention is usually expensive (the manufacturing line is stopped), thus the chance for false alarm is to be diminished:

$z_{\alpha/2}$ =3 (the so called $\pm 3\sigma$ limit),

then α=0.0027, that is the chance for erroneous decision is about three from among one thousand.

$$\underset{LCL}{\mu - 3\sigma/\sqrt{n}} < \bar{x} < \underset{UCL}{\mu + 3\sigma/\sqrt{n}}$$

The region of acceptance: $\mu - 3\sigma/\sqrt{n} < \bar{x} < \mu + 3\sigma/\sqrt{n}$

Problem 1

μ and σ are not known (we do not know the reference to which the process is to be compared)

→ estimation from a large sample

Problem 2

We may not be sure if the process used for estimating μ and σ is in control

→ check using control chart

Phase I: establishing stability and control limits

Phase II: on-going control using the previously established control limits

THE X-BAR – RANGE CHART

n (typically *n*=3 - 5) samples are taken from the process time to time. The mean and the range of the sample is computed:

$$R = \left| x_{\max} - x_{\min} \right| \qquad \bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$$

An R_i range and $\bar{x}_i$ mean is found for the sample *i*.

$$\hat{\mu} = \bar{\bar{x}} = \frac{1}{m}\sum_i \bar{x}_i$$

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \quad \text{where} \quad \bar{R} = \frac{1}{m}\sum_i R_i$$

CONSTRUCTION OF THE X-BAR CHART

Phase I

$$CL_{\bar{x}} = \bar{\bar{x}} = \frac{1}{m}\sum_i \bar{x}_i$$

(*m* is the number of samples, $\bar{x}_i$ is the mean of the *i*-th sample)

$$UCL_{\bar{x}} = \bar{\bar{x}} + \frac{3\bar{R}}{d_2\sqrt{n}} = \bar{\bar{x}} + A_2\bar{R}$$

(upper control limit)

$$LCL_{\bar{x}} = \bar{\bar{x}} - \frac{3\bar{R}}{d_2\sqrt{n}} = \bar{\bar{x}} - A_2\bar{R}$$

(lower control limit)

Phase II (on-going control)

$\bar{\bar{x}}$ and $\bar{R}$ from Phase I, that is the center line and control limits are given

CONSTRUCTION OF THE RANGE (R) CHART

$$\mathrm{H}_0 : Var(x) = \sigma_0^2$$

Phase I

$$CL_R = \overline{R} = \frac{1}{m}\sum_i R_i \qquad \hat{\sigma}_R = d_3\hat{\sigma} = \frac{d_3\overline{R}}{d_2} = \frac{(D_4 - 1)\overline{R}}{3}$$

The control limits for the ±3σ rule:

$$UCL_R = \overline{R} + 3\hat{\sigma}_R = \overline{R} + 3\frac{d_3\overline{R}}{d_2} = \overline{R}\left(1 + 3\frac{d_3}{d_2}\right) = D_4\overline{R}$$

$$LCL_R = \overline{R} - 3\hat{\sigma}_R = \overline{R} - 3\frac{d_3\overline{R}}{d_2} = \overline{R}\left(1 - 3\frac{d_3}{d_2}\right) = D_3\overline{R}$$

If negative value is obtained for *LCL*, it is to be set as zero

---

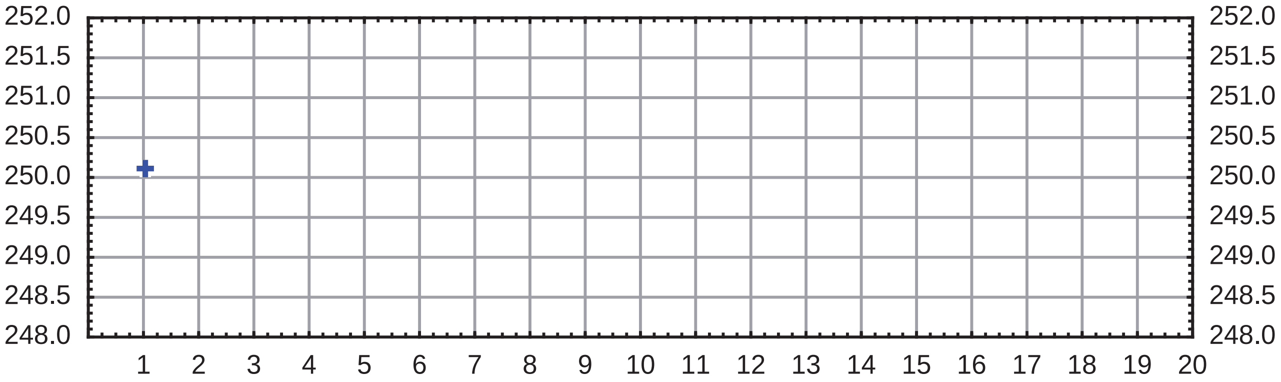
TABLE OF CONSTANTS

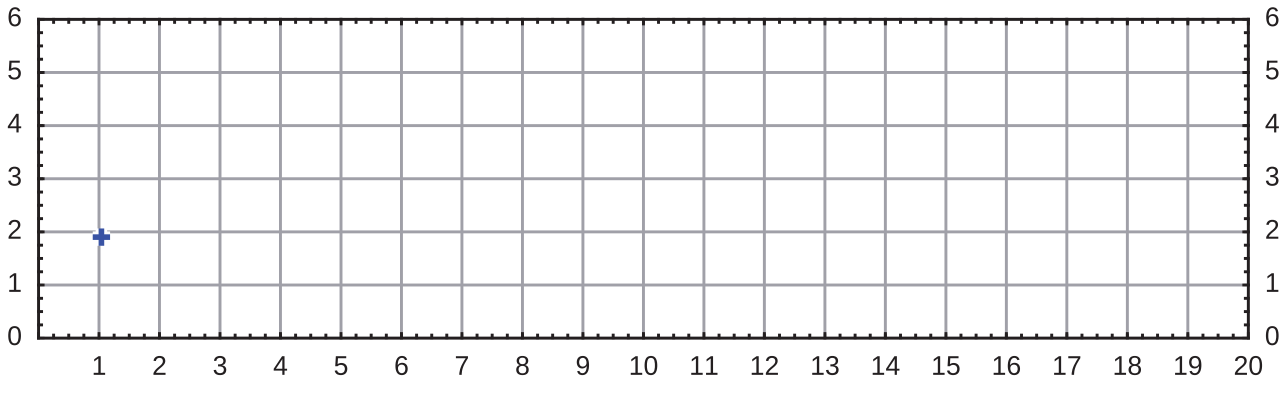
n	d_2	d_3	c_4	A_2	A_3	B_3	B_4	D_3	D_4
2	1.128	0.853	0.7979	1.880	2.659	0	3.267	0	3.267
3	1.693	0.886	0.8862	1.023	1.954	0	2.568	0	2.574
4	2.059	0.880	0.9213	0.729	1.628	0	2.266	0	2.282
5	2.326	0.864	0.9400	0.577	1.427	0	2.089	0	2.114

Example 2

Prepare an X-bar/R chart using the data in the table!

i	measured sample elements					mean	*R*
1	251.25	249.67	250.15	250.22	249.30	250.118	1.950
2	247.56	249.84	251.04	249.47	250.25		
3	251.47	250.23	250.07	250.12	250.37		
4	249.35	249.77	249.29	250.92	250.44	249.954	1.630
5	249.09	251.09	248.14	248.51	250.90	249.546	2.950
6	251.59	248.13	250.06	248.92	252.09	250.158	3.960
7	250.61	249.55	249.23	249.61	251.39	250.078	2.160
8	249.95	247.74	249.40	248.88	249.16	249.026	2.210
9	247.74	249.42	249.59	251.59	250.36	249.740	3.850
10	247.89	250.65	249.61	249.08	248.72	249.190	2.760
11	249.26	250.08	251.22	250.08	250.26	250.180	1.960
12	249.83	249.46	248.83	251.56	249.16	249.768	2.730
13	250.36	250.10	251.68	250.36	248.78	250.256	2.900
14	250.71	250.26	250.18	249.47	250.72	250.268	1.250
15	250.50	252.36	251.52	249.91	250.75	251.008	2.450
16	250.11	250.87	249.31	249.93	249.63	249.970	1.560
17	248.81	249.65	248.08	250.57	251.48	249.718	3.400
18	249.90	249.81	250.59	250.38	250.74	250.284	0.930
19	250.88	249.79	249.85	250.11	250.61	250.248	1.090
20	249.27	248.61	250.64	249.43	249.60	249.510	2.030
mean						249.955	2.333

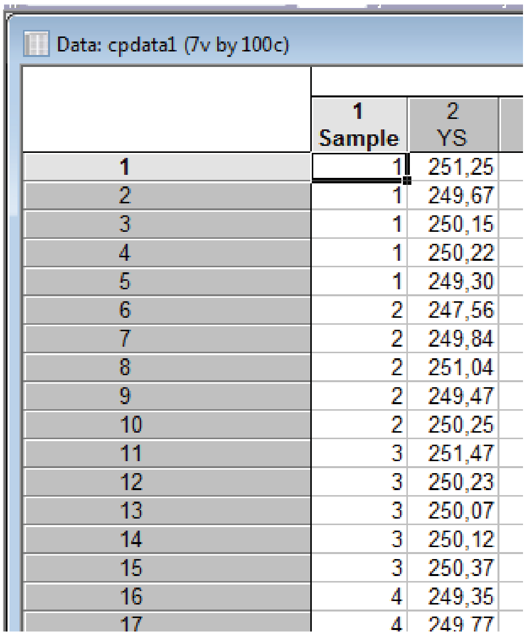




Example 3

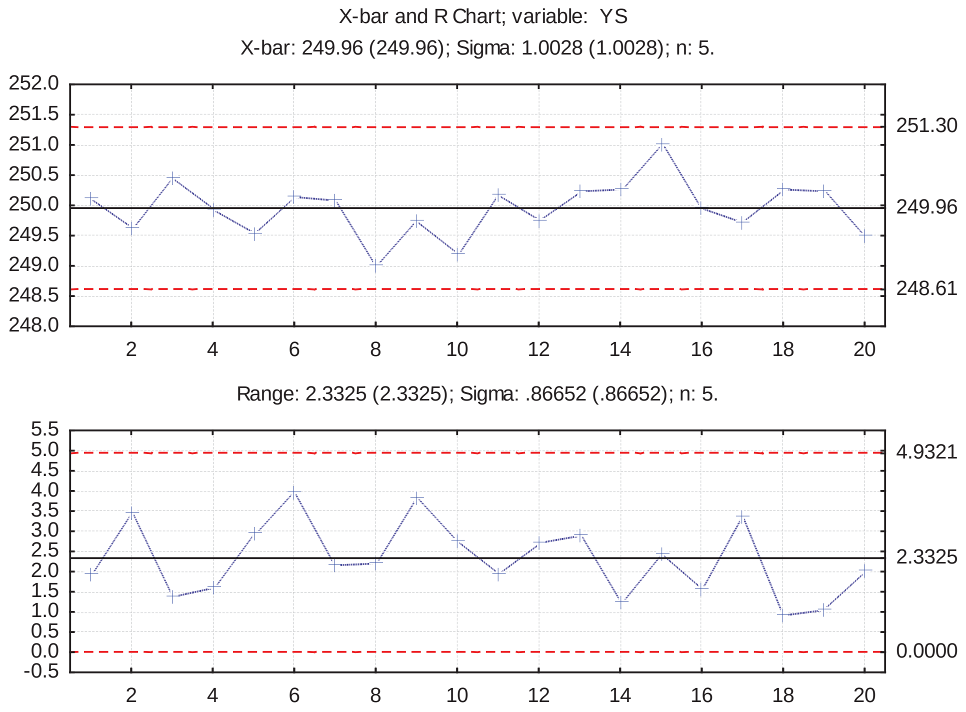
Prepare an X-bar/R chart using the YS column of the cpdata1.sta data file!

Phase I or Phase II?

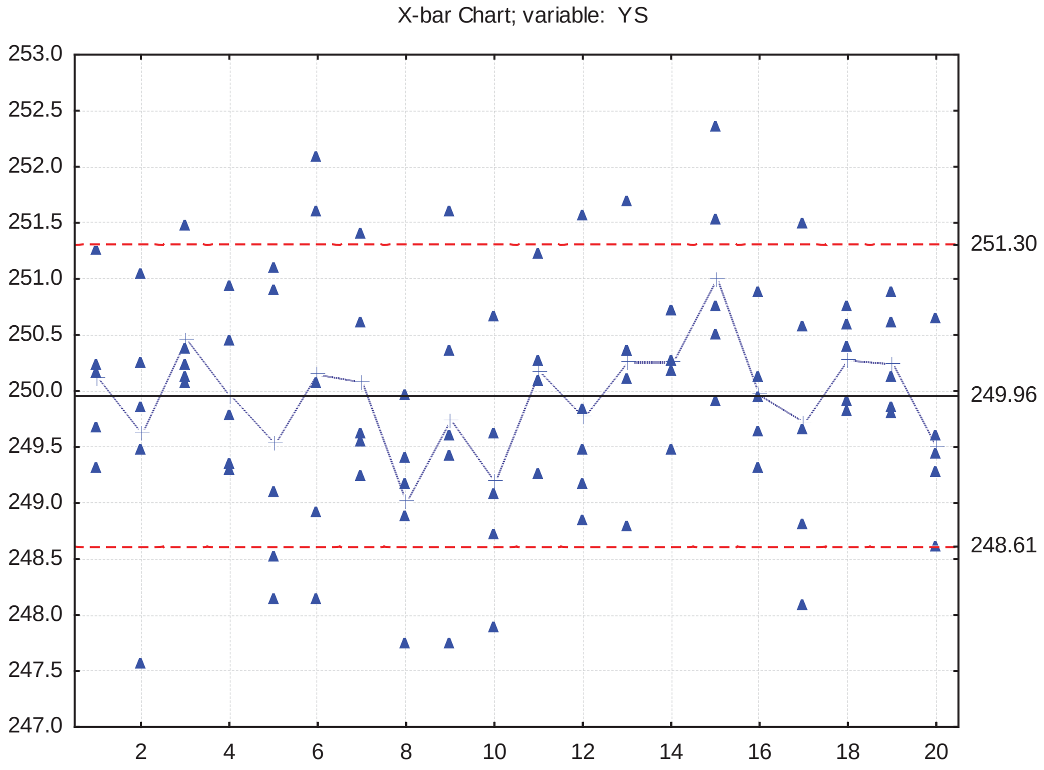


Data: cpdata1 (7v by 100c)

	1 Sample	2 YS
1	1	251,25
2	1	249,67
3	1	250,15
4	1	250,22
5	1	249,30
6	2	247,56
7	2	249,84
8	2	251,04
9	2	249,47
10	2	250,25
11	3	251,47
12	3	250,23
13	3	250,07
14	3	250,12
15	3	250,37
16	4	249,35
17	4	249,77

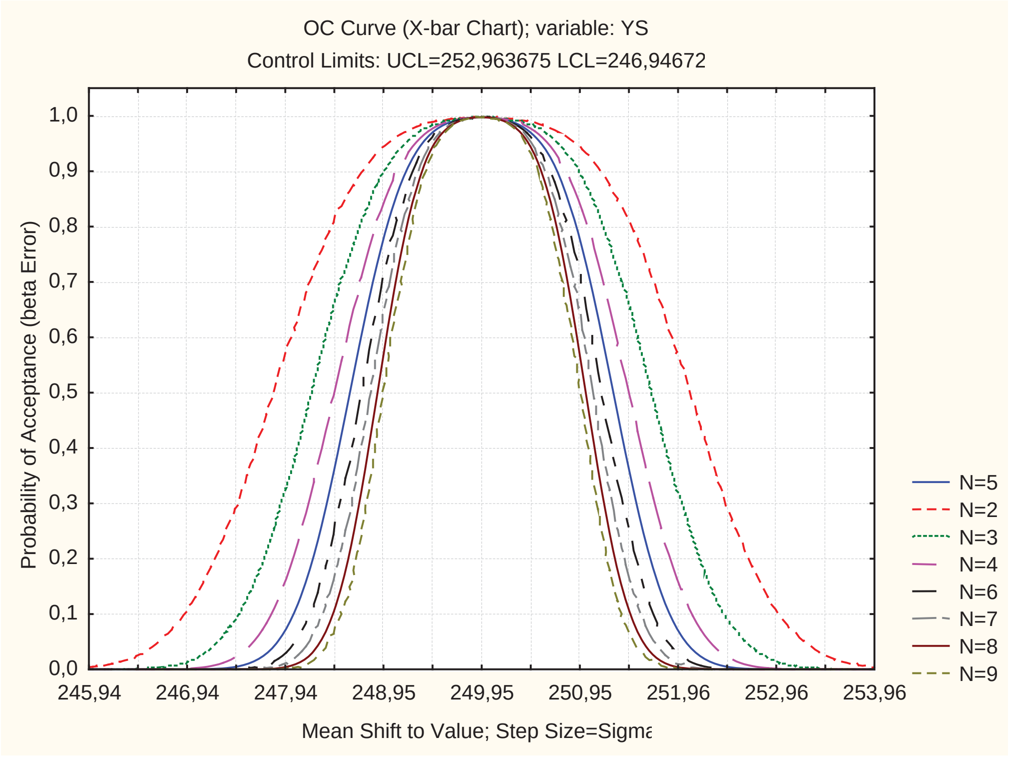


The control limits on the X-bar chart refer to the mean, not to single measurement values!



OPERATING CHARACTERISTIC CURVE

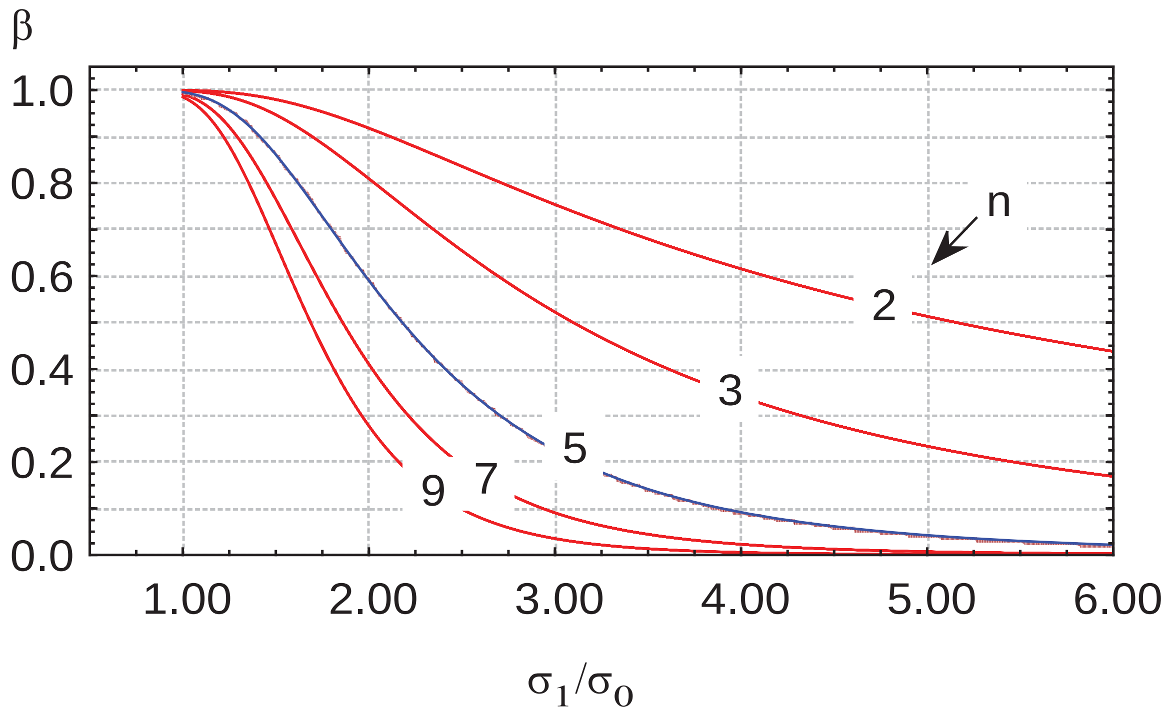
Operating Characteristic (OC) curve for the X-bar chart (α =0.0027)



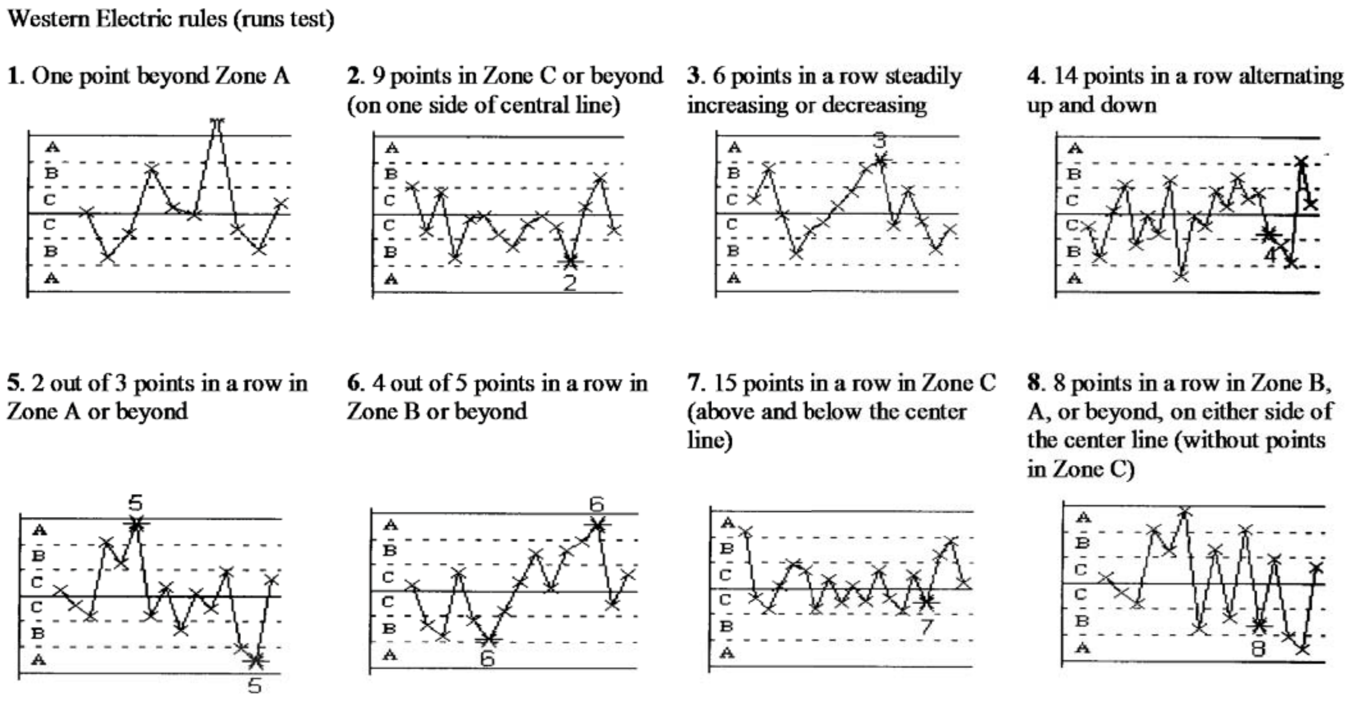
The true mean is on the horizontal axis

Operating Characteristic Curve

Operating Characteristic (OC) curve for the R chart (±3σ, that is α=0.0027)



THE WESTERN ELECTRIC ALGORITHMIC RULES (RUNS TESTS)



Western Electric rules (runs test)

1. One point beyond Zone A

2. 9 points in Zone C or beyond (on one side of central line)

3. 6 points in a row steadily increasing or decreasing

4. 14 points in a row alternating up and down

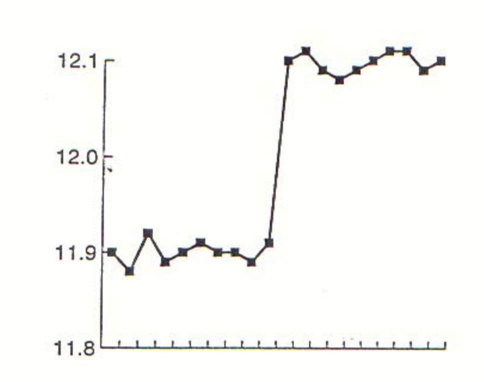
5. 2 out of 3 points in a row in Zone A or beyond

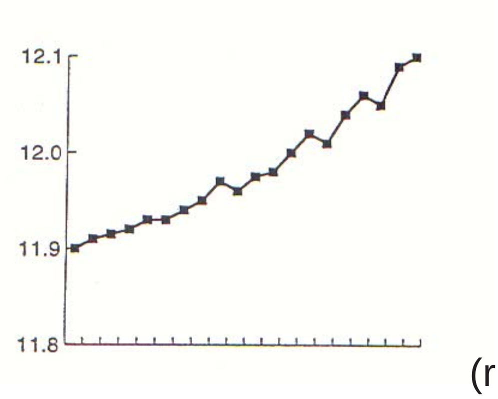
6. 4 out of 5 points in a row in Zone B or beyond

7. 15 points in a row in Zone C (above and below the center line)

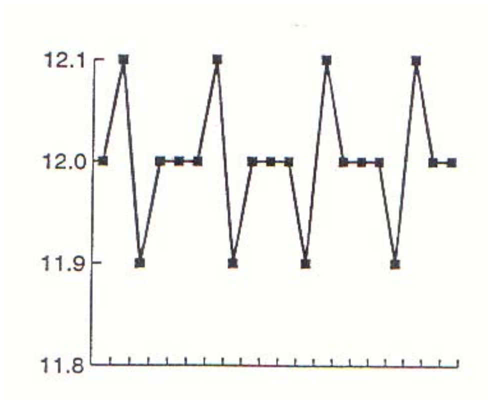
8. 8 points in a row in Zone B, A, or beyond, on either side of the center line (without points in Zone C)

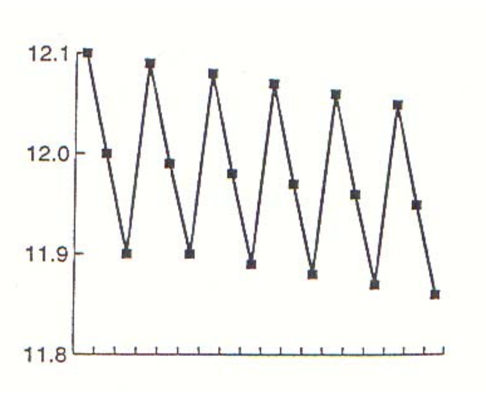
THE USE OF PLOTTING DATA





(run charts)





WHEN TO USE X-BAR CHART?

- if subgroups (at similar conditions) may be drawn from the process;
- if large ($\Delta \geq 2\sigma$) deviations are expected, and these are to be detected;
- if small deviations do not cause serious economic consequences;
- if the simplicity of the procedure is a point, but computation of sample mean is feasible;
- the cost of sampling is relatively low.

WHEN NOT TO USE X-BAR CHART?

- if subgroups (at similar conditions) may not be drawn from the process;
- if the within-groups fluctuation is much smaller than the between-groups fluctuation, since in this case many outliers were obtained;
- if the deviation to be detected is in the range $0.5\sigma<\Delta<2\sigma$;
- if the cost of sampling/analysis is higher than could be gained by control;
- the process inherently cyclic or it contains trend, in that case the consecutive samples are not independent.

STEPS FOR PREPARING AND APPLYING THE X-BAR/R-CHART

- Variable selection is relevant for quality, the measurement should not cost more than omitting the control.
- Deciding on rational subgroups: items produced under essentially the same conditions: the within-subgroup variation should be much less than the fluctuation between subgroups, when possible, consecutive units are used.
- Preliminary estimation of the fluctuation parameter for the process (σ^2) in order to decide the subgroup size; range is used for $n<10$. The subgroup size is usually 4-6, 5 is typical.

STEPS FOR PREPARING AND APPLYING THE X-BAR/R-CHART

- Phase I: Data collection for estimating process parameters (μ and σ^2)
  Usually 25 subgroups are taken, plotting the data on charts (location and spread), computation of center line and control limits (trial control limits).

- Deciding on stability (control)
  If instability occurs, the special causes are found and eliminated. The belonging points are scratched, control limits are recalculated. This procedure is repeated until stability is achieved, additional samples may be drawn if required. This is the end of Phase I.

STEPS FOR PREPARING AND APPLYING THE X-BAR/R-CHART

- On-going control (Phase II) is started if the process is proved to be in control. The analysis is started with the chart of fluctuation (e.g. range) because the control limits of the X-bar chart are valid only for s =const case. If an outlier occurs, printing error is assumed first (its detection is cheap). The on-going control is to be performed real-time, it has not much sense to discover the necessity of an action for the previous day.

CONTROL CHARTS FOR INDIVIDUAL VALUES

It is not feasible to use averages and ranges:

- the production rate is too slow
- the output is too homogeneous over short time intervals (e.g. concentration of a solution).

Individual value (I or X) chart center line and control limits:

$$CL_x = \bar{x} \qquad UCL_x = \bar{x} + \frac{3\overline{MR}}{d_2} \qquad LCL_x = \bar{x} - \frac{3\overline{MR}}{d_2}$$

MR: Moving Range

$$MR_i = \left| x_i - x_{i-1} \right| \qquad \overline{MR} = \frac{\sum_{i=2}^{m} MR_i}{m-1} \qquad \hat{\sigma} = \frac{\overline{MR}}{d_2}$$

MOVING RANGE (MR) CHART

Center line and control limits:

$$CL_{MR} = \overline{MR} \qquad UCL_R = \bar{R} + 3\hat{\sigma}_R = \bar{R} + 3\frac{d_3\bar{R}}{d_2} = D_4\bar{R}$$

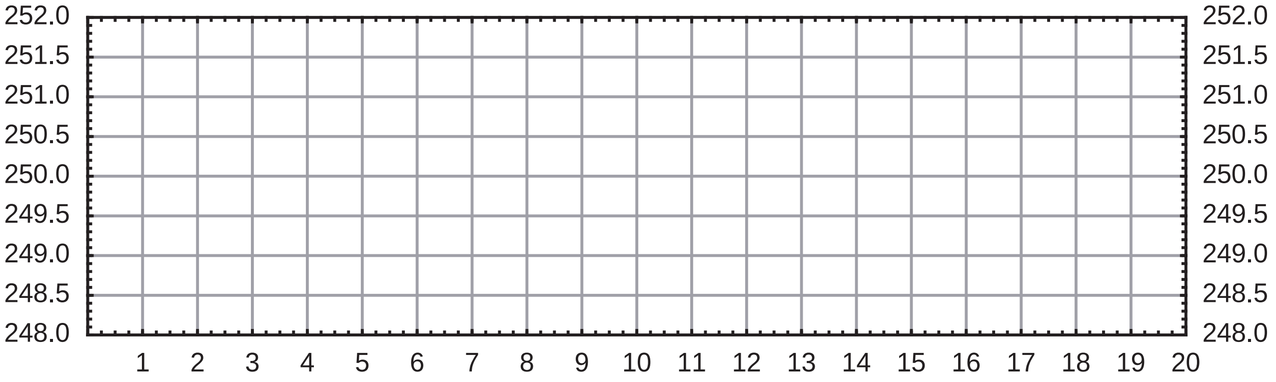
$$UCL_{MR} = D_4\overline{MR}$$

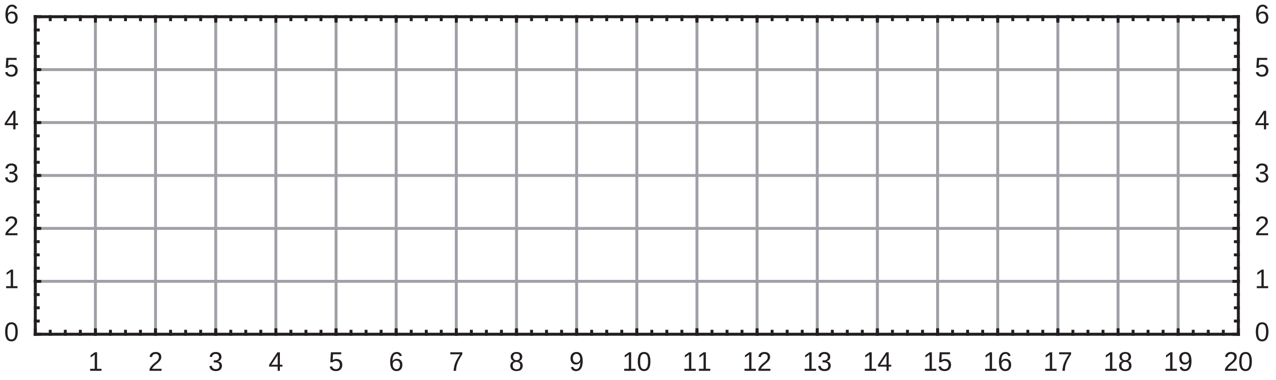
$$LCL_{MR} = D_3\overline{MR}$$

Example 4

Prepare an individual value and moving range chart from the data in the table!

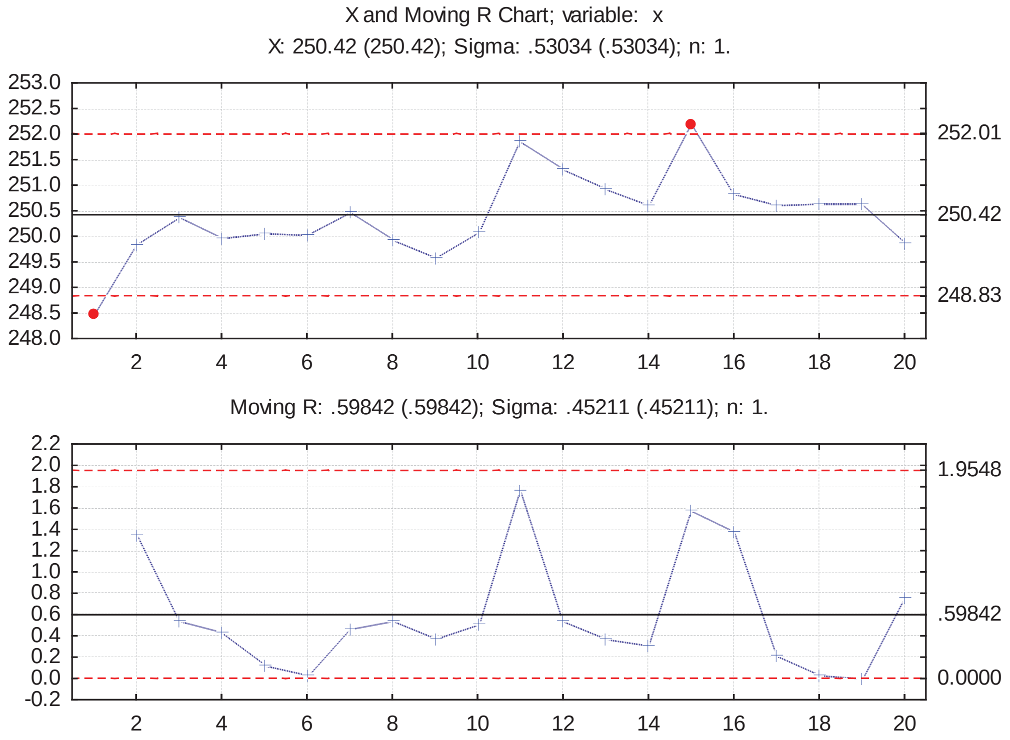
	x_i	$MR_i = \left|x_i - x_{i-1}\right|$
1	248.49	-
2	249.84	1.35
3	250.39	
4	249.96	
5	250.08	
6	250.04	
7	250.50	0.46
8	249.95	0.55
9	249.57	0.38
10	250.09	0.52
11	251.86	1.77
12	251.32	0.54
13	250.94	0.38
14	250.63	0.31
15	252.21	1.58
16	250.83	1.38
17	250.61	0.22
18	250.64	0.03
19	250.64	0.00
20	249.88	0.76
average	250.4235	0.5984





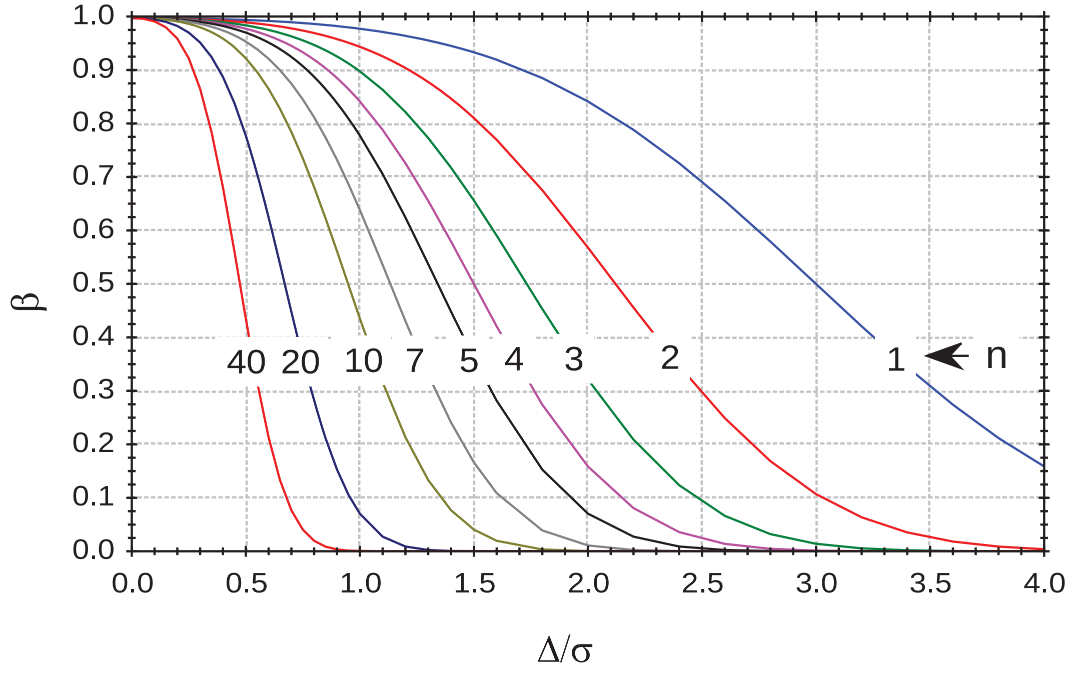
Example 5

Prepare an individual value and moving range chart from the data in the Indiv1.xls!



OPERATING CHARACTERISTIC CURVE

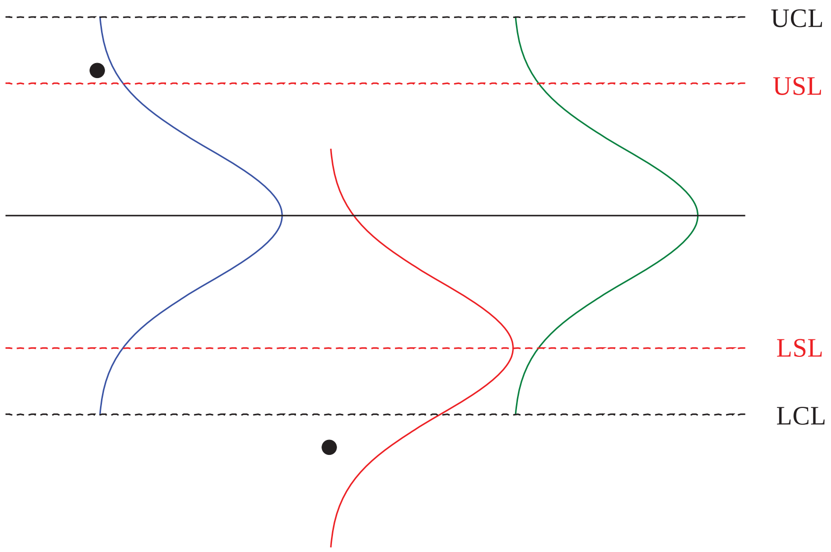
Operating Characteristic (OC) curve for the X-bar chart (α=0.0027)



SUMMARY TABLE FOR THE VARIABLES CONTROL CHARTS

Type of the chart			
$\overline{x}-R$	$\overline{x}-s$	$\overline{x}-s^2$	x–MR
$CL_{\overline{x}}=\overline{\overline{x}}$	$CL_{\overline{x}}=\overline{\overline{x}}$	$CL_{\overline{x}}=\overline{\overline{x}}$	$CL_x=\overline{x}$
$UCL_{\overline{x}}=\overline{\overline{x}}+\frac{3\overline{R}}{d_2\sqrt{n}}=\overline{\overline{x}}+A_2\overline{R}$	$UCL_{\overline{x}}=\overline{\overline{x}}+3\frac{\overline{s}}{c_4\sqrt{n}}=\overline{\overline{x}}+A_3\overline{s}$	$UCL_{\overline{x}}=\overline{\overline{x}}+3\frac{\sqrt{\overline{s^2}}}{\sqrt{n}}$	$UCL_x=\overline{x}+\frac{3\overline{MR}}{d_2}$
$LCL_{\overline{x}}=\overline{\overline{x}}-\frac{3\overline{R}}{d_2\sqrt{n}}=\overline{\overline{x}}-A_2\overline{R}$	$LCL_{\overline{x}}=\overline{\overline{x}}-3\frac{\overline{s}}{c_4\sqrt{n}}=\overline{\overline{x}}-A_3\overline{s}$	$LCL_{\overline{x}}=\overline{\overline{x}}-3\frac{\sqrt{\overline{s^2}}}{\sqrt{n}}$	$LCL_x=\overline{x}-\frac{3\overline{MR}}{d_2}$
$CL_R=\overline{R}$	$CL_s=\overline{s}$	$CL_{s^2}=\overline{s^2}$	$CL_{MR}=\overline{MR}$
$UCL_R=\overline{R}+3\frac{d_3\overline{R}}{d_2}=D_4\overline{R}$	$UCL_s=\overline{s}+3\frac{\overline{s}}{c_4}\sqrt{1-c_4^2}=B_4\overline{s}$	$UCL_{s^2}=\frac{\overline{s^2}\chi^2_{fölsô}}{\nu}$	$UCL_{MR}=D_4\overline{MR}$
$LCL_R=\overline{R}-3\frac{d_3\overline{R}}{d_2}=D_3\overline{R}$	$LCL_s=\overline{s}-3\frac{\overline{s}}{c_4}\sqrt{1-c_4^2}=B_3\overline{s}$	$LCL_{s^2}=\frac{\overline{s^2}\chi^2_{alsó}}{\nu}$	$LCL_{MR}=D_3\overline{MR}$

WHY NOT THE SPECIFICATION LIMITS ARE USED IN THE CHART?



a)

WHY NOT THE SPECIFICATION LIMITS ARE USED IN THE CHART?